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Q What are the power in A.C circuit. Derive expression for the average power in an A.C circuit and define power factor and wattless current?

Ans Power in A.C circuit :→

In general, the rate of doing work is called power. The power in an electric circuit is the rate in which electrical energy is consumed in the circuit. As in A.C circuit both the applied e.m.f and current vary continuously with time, hence the power in an A.C is equal to the product of the instantaneous e.m.f & instantaneous current averaged over a complete cycle. The power of an A.C circuit depends on the fact whether a phase difference between the current and e.m.f exist or not.

In an A.C circuit the instantaneous values of the voltage and current are given by

$E = E_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \theta)$

Hence the power in an A.C circuit at any instant

$$= EI$$

$$= E_0 \sin\omega t \times I_0 \sin(\omega t - \theta)$$

$$= E_0 I_0 \sin\omega t(\sin\omega t \cdot \cos\theta - \cos\omega t \cdot \sin\theta)$$

$$= E_0 I_0 [\sin^2\omega t \cos\theta - \sin\omega t \cdot \cos\omega t \cdot \sin\theta]$$

$$= E_0 I_0 \left[\tfrac{1}{2}(1 - \cos 2\omega t)\cos\theta - \tfrac{1}{2}\sin 2\omega t \cdot \sin\theta\right]$$

$$= \frac{E_0 I_0}{2}[\cos\theta - \cos 2\omega t \cdot \cos\theta - \sin 2\omega t \cdot \sin\theta]$$

$$= \frac{E_0 I_0}{2}[\cos\theta - \cos(2\omega t - \theta)]$$

This shows that the power consumed also varies with time. Hence the average power P during each complete cycle is given by

$$P = \frac{1}{T}\left[\int_0^T EI\,dt\right]$$

$$= \frac{1}{T} \times \frac{1}{2} E_0 I_0 \int_0^T [\cos\theta - \cos(2\omega t - \theta)]\,dt$$

$$= \frac{E_0 I_0}{2T}\left[\int_0^T \cos\theta\,dt - \int_0^T \cos(2\omega t - \theta)\,dt\right.$$

$$= \frac{E_0 I_0}{2T}\left[\cos\theta\, t - \frac{\sin(2\omega t-\theta)}{2\omega}\right]_0^T$$

$$= \frac{E_0 I_0}{2T}\left[\cos\theta \cdot T - 0 - \frac{\sin(2\omega T-\theta)}{2\omega} + \frac{\sin(-\theta)}{2\omega}\right]$$

$$\because T = \frac{2\pi}{\omega}$$

$$\therefore P = \frac{E_0 I_0}{2\times\frac{2\pi}{\omega}}\left[\cos\theta\times\frac{2\pi}{\omega} - \frac{\sin(4\pi-\theta)}{2\omega} + \frac{\sin(-\theta)}{2\omega}\right]$$

$$= \frac{E_0 I_0}{2}\cos\theta \quad [\because \sin(4\pi-\theta) = \sin(-\theta)]$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos\theta$$

$$\therefore P = E_{r.m.s} \times I_{r.m.s} \times \cos\theta$$

$\therefore$ Average power = Virtual power $\times \cos\theta$

As $\cos\theta$ is the factor by which the product of the r.m.s values of the voltage and current must be give the power dissipated. It is known as power factor.

Watless current :→

The current in A.C circuit is said to be wattless when the average power consumed in the circuit is zero. This is possible when the power factor is zero

$\therefore \cos\theta = 0 \quad \therefore \theta = \pi/2$

when the current and voltage differ in phase by 90°. For example, In pure inductance, the current lags behind the voltage by $\pi/2$, the <P> absorbed in pure inductance is zero. Hence power is absorbed in the magnetic field of the coil during the first quarter cycle, but is returned back to the generator during the next quarter cycle.

In the similar way, the average power <P> absorbed in a pure capacitance is also zero.